Accurate Delay Distribution for IEEE 802.11 DCF

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***Abstract*— We derive the access delay generating function of the IEEE 802.11 DCF protocol. Our analysis corrects a model recently presented in [6]. We demonstrate that numerical transform inversion can be used to efficiently obtain values of the distribution. Simulations show that our analytical results are highly accurate.**

***Index Terms*— IEEE 802.11 DCF, MAC access delay, generating function, numerical inversion.**

I. Introduction

CURRENTLY, analytical modeling and performance evaluation of the IEEE 802.11 Medium Access Control (MAC) protocol [4] for wireless local area networks is an area of active research. The MAC layer employs a channel access mechanism called the distributed coordination function (DCF) where stations contend for the channel using a carrier sense multiple access mechanism with collision avoidance (CSMA/CA).

In this paper, we are concerned with the access delay of the IEEE 802.11 DCF. We define the access delay as the time interval between the instant when the packet reaches the head of the transmission queue and begins contending for the channel, and the instant when the packet is successfully received at the destination station. A trivial variation on the access delay is the service-time delay which equals the access delay plus the (deterministic) time to receive an acknowledgement packet. Existing work on the delay performance of DCF has focused primarily on the average access delay, with most papers using the seminal Markov chain model developed in [1] as a starting point. Recently, the authors in [7] proposed a method to derive the service-time delay generating function from the same Markov chain model. However, this approach is rather complex and, as we will demonstrate, the distributional values predicted by its generating function are not very accurate. A new approach for deriving the service-time delay generating function is presented in [6]; however the paper ignored an important detail, namely the dependence between the number of backoff slots of a node and the delay due to transmissions and collisions of competing stations. Neither [7] nor [6] discuss the non-trivial task of deriving the distribution from the generating function. In this letter, we develop the access delay generating function taking into account the dependency mentioned above, and show that the delay distribution can be readily computed by numerically inverting the generating function.

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II. Analytical Model for the Access Delay

We consider N stations that always have packets to send (saturated stations), and assume ideal channel conditions so that the only source of packet corruption is packet collision. We assume as in [6] that each packet regardless of its source collides with a constant and independent probability p. Based on the mechanism associated with the exponential backoff of DCF protocol, the overall average backoff window can be calculated. In particular, a packet is successfully transmitted with probability $1-p$, and the average backoff for such packet is $(W-1)/2$, where W is an initial backoff window. If the first transmission fails, the packet is successfully transmitted on the second attempt with probability $(1-p)p$, and the average backoff in this case is $(2W-1)/2$. Using similar arguments, the overall average backoff window is then given by

$$\overline{W} = \eta \left(\sum_{i=0}^{m-1} p^i \; (2^i W - 1)/2 + \sum_{i=m}^{K-1} p^i \; (2^m W - 1)/2 \right), \tag{1}$$

where m is the number of times the backoff window is doubled, $K \geq m$ is the maximum number of transmissions for one packet, $\eta = (1-p)/(1-p^K)$ and $(1-p^K)^{-1}$ is a normalization term. The collision probability p can be expressed as a function of the $\overline{W}$ as follows [6]

$$p = 1 - (1 - 1/\overline{W})^{N-1}. \tag{2}$$

Equations (1) and (2) establish a fixed point formulation from which the collision probability p can be computed using an iterative method.

Next we derive the generating function of the access delay using a similar approach to [6] by decomposing the delay into different components. We define the following discrete random variables (RVs) and discrete probability density functions (pdfs). Let X, Y, V be RVs representing, respectively, the number of backoff slots of the tagged station, the delay contribution due to packet transmissions and the collisions of the tagged station, and the delay contribution due to packet transmissions and collisions of other stations. Furthermore, let

- $u_i()$ denote the discrete uniform pdf between 0 and $2^i W - 1, \; 0 \leq i \leq m$,
- $l()$ denote the pdf of the RV representing the channel occupancy of a transmitted packet,
- $\psi()$ denote the pdf of the RV representing the channel occupancy of collisions involving the tagged station,
- $\theta()$ denote the pdf of the RV representing the channel occupancy of a collision not involving the tagged station,
- $f^{(i)}()$ denote the i-fold convolution of the pdf $f()$,

- $F(z)$ denote the Z-transform of the pdf $f()$, e.g. $\Psi(z), \Theta(z)$ are the Z-transform of the above $\psi(), \theta()$ functions, respectively.

We first derive the probability of delay due to backoffs, transmissions and collisions that involve the tagged station. With normalized probability η, the packet is successfully transmitted in its first attempt after x backoff slots. Since the number of backoff slots is chosen uniformly between 0 and $W-1$, the probability that there are x backoff slots is $u_0(x)$. The packet transmission time takes y units of time with probability $l(y)$. Thus, $P(X=x, Y=y) = \eta\ u_0(x)\ l(y)$. With probability ηp the first transmission attempt fails, but the second attempt is successful. In this case, the probability that there are x backoff slots is $u_0 * u_1(x)$ where $*$ represents convolution. The y units of time delay caused by the first collision and the packet transmission time in the second attempt has probability $\psi * l(y)$. Extending this logic to $K-1$ retransmissions, the probability that the tagged station experiences x backoff slots, and that there are y units of time delay due to collisions and packet transmission involving the tagged station, is given by

$$\begin{aligned} &P(X=x, Y=y) = \eta\ u_0(x)\ l(y) \\ &+ \eta\ p\ u_0 * u_1(x)\ \psi * l(y) + ... \\ &+ \eta\ p^m\ u_0 * u_1 * \ ..\ * u_m(x)\ \psi^{(m)} * l(y) + ... \\ &+ \eta\ p^{K-1}\ u_0 * \ ..\ * u_m^{(K-m)}(x)\ \psi^{(K-1)} * l(y). \end{aligned} \quad (3)$$

In the rest of the paper we use the shorthand notation $P(x, y)$ for $P(X=x, Y=y)$, etc..

The backoff period of the tagged station can be interrupted by transmissions and/or collisions of other stations. The probability that an arbitrary backoff slot of the tagged station is interrupted by one or more other stations is $q = 1 - (1 - 1/\overline{W})^{N-1}$. The probability that an arbitrary backoff slot of the tagged station is interrupted by only one other station is $q' = \binom{N-1}{1}(1/\overline{W})(1-1/\overline{W})^{N-2}$. Thus, the probability that there is a collision, given that a slot is interrupted, can be expressed as

$$P(\text{collision}|\text{slot is interrupted}) = q_c = (q - q')/q. \quad (4)$$

Given that the tagged station experiences x backoff slots before it successfully transmits a packet, the probability that the transmissions and collisions of other stations during these backoff slots contribute v units of time to the tagged station's delay is

$$\begin{aligned} P(v|x) = &\sum_{j=0}^{x} \binom{x}{j} q^j (1-q)^{x-j} \\ &\times \sum_{i=0}^{j} \binom{j}{i} q_c^i (1-q_c)^{j-i} l^{(j-i)} * \theta^{(i)}(v). \end{aligned} \quad (5)$$

Equation (5) expresses the dependence between the number of backoff slots and the number of transmissions and collisions of other stations during this period. In particular, given x backoff slots, there are j interruptions by other stations and among them i interruptions result in collision. The authors in [6] ignored this dependence i.e. they made the approximation that the probability that there are j interruptions by other stations is independent of the number of backoff slots of the tagged station (see (17), (31) in [6]). This approximation leads to significant errors in the distributional values, as we illustrate in Section III.

To calculate the generating function, let k be an integer, $k \geq 0$, and let $g(k) = \sum_{\Omega} P(x, y, v)$, where $\Omega = \{(x, y, v) : \delta x + y + v = k\}$, and δ is the length of the backoff slot in units of time. The generating function of the access delay of the tagged station can be expressed as:

$$\begin{aligned} G(z) = \sum_{k=0}^{\infty} g(k) z^k &= \sum_{\delta x + y + v = 0}^{\infty} P(x, y, v) z^{\delta x + y + v} \\ &= \sum_{\delta x + y = 0}^{\infty} P(x, y) z^{\delta x + y} \sum_{v=0}^{\infty} P(v|x, y) z^v, \end{aligned} \quad (6)$$

where the notation $\sum_{\delta x + y + v = k}, 0 \leq \forall k \leq \infty$ is to be understood as the sum over all combinations of x, y, v nonnegative integers such that $\delta x + y + v = k$. Observe that $P(v|x, y) = P(v|x)$ and the second term of the product in (6) can be simplified using (5) (after some algebraic manipulations) as follows:

$$\begin{aligned} \sum_{v=0}^{\infty} P(v|x) z^v = &\sum_{j=0}^{x} \binom{x}{j} q^j (1-q)^{x-j} \\ &\times \sum_{i=0}^{j} \binom{j}{i} q_c^i (1-q_c)^{j-i} L^{j-i}(z) \Theta^i(z) = A^x(z), \end{aligned}$$

where $A(z) = q(q_c \Theta(z) + (1-q_c) L(z)) + (1-q)$.

Substituting back into (6), we have

$$\begin{aligned} G(z) &= \eta\ L(z)\ \sum_{i=0}^{m-1} p^i\ \Psi^i(z) \prod_{j=0}^{i} U_j(\hat{z}) \\ &+ \eta\ L(z)\ p^m\ \Psi^m(z) \prod_{j=0}^{m} U_j(\hat{z}) \\ &\times \frac{1 - [p\ U_m(\hat{z}) \Psi(z)]^{K-m}}{1 - p\ U_m(\hat{z}) \Psi(z)}, \end{aligned} \quad (7)$$

where $\hat{z} = z^{\delta} A(z)$, and $\forall\ i = 0, 1, .., m$, $U_i(\hat{z}) = (1/(2^i W))(1 - \hat{z}^{2^i W})/(1 - \hat{z})$.

To numerically invert the generating function, we use the lattice-poisson algorithm developed in [3]. The inversion formula used in this algorithm is

$$g(k) \approx \frac{1}{2klr^k} \sum_{j=-kl}^{kl-1} G(re^{-i\pi j/(kl)}) e^{i\pi j/l},$$

for real r and integer l. The results we present in the next section are calculated using $l = 1$ and $r = 10^{-4/k}$, which results in an error less than 10^{-8} in the numerical inversion process.

III. Numerical Evaluation and Discussion

In this section we verify our analytical results by simulation, and compare the delay distribution of our model with those obtained by inverting the generating functions given in [6], [7] using the lattice-poisson algorithm. (To be precise, we invert the access delay variants of the service-time generating functions that appear in these papers.) The simulation is conducted using the ns-2 simulator (version 2.27) [2]. We found that the MAC implementation in ns-2 simulator contains several bugs that noticeably affect the output delay statistics, so these were remedied. The main problems with the standard simulator are: the timer modelling the DIFS (distributed interframe space)

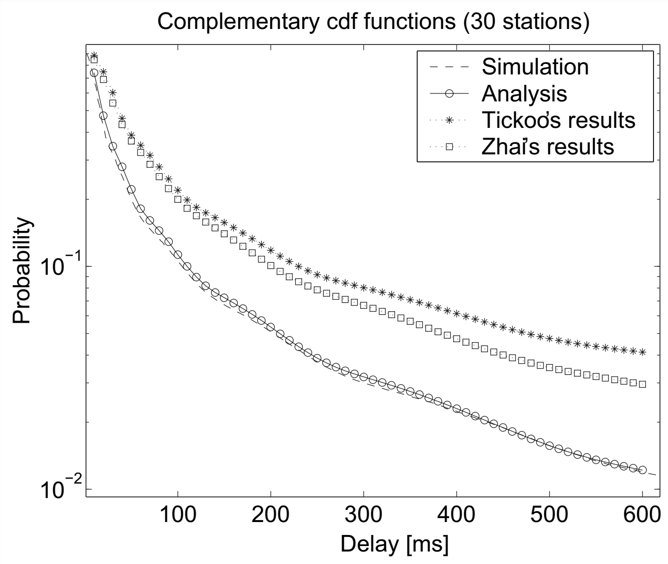

Fig. 1. Access delay distribution (ccdf) function for 30 stations using packet size of 1000 bytes.

deferral is not stopped when the channel becomes busy; a post-backoff is not preceded by a DIFS; after the backoff counter is frozen, the remaining backoff time is incorrectly calculated; and the EIFS (extended interframe space) period is erroneously followed by a DIFS deferral.

We simulate the access delay on the uplink where a number of mobile stations send to an access point. In our setup the four-way handshake (RTS/CTS) mechanism for channel reservation is not used. All the stations are saturated using UDP traffic with the same fixed packet size, and consequently collisions are also then of fixed duration. Note that although the results presented here use a fixed UDP packet size, our analytical model is valid for general packet size distribution. We simulate an 802.11b system [5], where the data transmission rate is 11 Mbps, the control rate is 1 Mbps, and m, K are set to 5 and 7, respectively. The initial window W is set to 32. Fig. 1 shows the delay distribution for $N = 30$ active stations and a packet size of 1000 bytes.

Observe that our analytical results exhibit excellent agreement with the simulation results. The delay distribution obtained from the generating function presented in [6] by Tickoo $et\ al.$ is inaccurate due to the reasons cited in Section II. Results obtained by using Zhai's $et\ al.$ generating function [7] are closer to the simulation results but still inaccurate. The inaccuracy stems from the model used for the delay contribution of non-tagged stations.

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